

Perturbative superluminal censorship and the null energy condition

Matt Visser,[†] Bruce Bassett,^{¶,*} and Stefano Liberati^{§,‡}

[†]*Physics Department, Washington University, Saint Louis, Missouri 63130-4899, USA*

^{¶,§}*International School for Advanced Studies (SISSA), Via Beirut 2-4, 34014 Trieste, Italy*

^{*}*Department of Theoretical Physics, University of Oxford, 1 Keble Road, OX1 3NP, UK*

[‡]*Istituto Nazionale di Fisica Nucleare (INFN), sezione di Trieste, Italy*

Abstract. We argue that “effective” superluminal travel, potentially caused by the tipping over of light cones in Einstein gravity, is always associated with violations of the null energy condition (NEC). This is most easily seen by working perturbatively around Minkowski spacetime, where we use linearized Einstein gravity to show that the NEC forces the light cones to contract (narrow). Given the NEC, the Shapiro time delay in any weak gravitational field is always a delay relative to the Minkowski background, and never an advance. Furthermore, any object travelling within the lightcones of the weak gravitational field is similarly delayed with respect to the minimum traversal time possible in the background Minkowski geometry.

INTRODUCTION

The relationship between the causal aspects of spacetime and the stress-energy of the matter that generates the geometry is a deep and subtle one. In this note, which is a simplified presentation based on our earlier work [1], we shall focus in somewhat more detail on the perturbative investigation of the connection between the null energy condition (NEC) and the light-cone structure. We shall demonstrate that in linearized gravity the NEC always forces the light cones to contract (narrow): Thus the validity of the NEC for ordinary matter implies that in weak gravitational fields the Shapiro time delay is always a delay rather than an advance.

This simple observation has implications for the physics of (effective) faster-than-light (FTL) travel via “warp drive”. It is well established, via a number of rigorous theorems, that any possibility of effective FTL travel via traversable wormholes necessarily involves NEC violations [2–5]. On the other hand, for effective FTL travel via warp drive (for example, via the Alcubierre warp bubble [6], or the Krasnikov FTL hyper-tube [7]) NEC violations are observed in

specific examples but it is difficult to prove a really general theorem guaranteeing that FTL travel implies NEC violations [1]. Part of the problem arises in even defining what we mean by FTL, and recent progress in this regard is reported in [1,8].

In this note we shall (for pedagogical reasons) restrict attention to weak gravitational fields and work perturbatively around flat Minkowski spacetime. One advantage of doing so is that the background Minkowski spacetime provides an unambiguous definition of FTL travel. A second advantage is that the linearized Einstein equations are simply (if formally) solved via the gravitational Liénard–Wiechert potentials. The resulting expression for the metric perturbation provides information about the manner in which light cones are perturbed.

LINEARIZED GRAVITY

For a weak gravitational field, linearized around flat Minkowski spacetime, we can in the usual fashion write the metric as [4,9,10]

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad (1)$$

with $h_{\mu\nu} \ll 1$. Then adopting the Hilbert–Lorentz gauge (*aka* Einstein gauge, harmonic gauge, de Donder gauge, Fock gauge)

$$\partial_\nu \left[h^{\mu\nu} - \frac{1}{2} \eta^{\mu\nu} h \right] = 0, \quad (2)$$

the linearized Einstein equations are [4,9,10]

$$\Delta h_{\mu\nu} = -16\pi G \left[T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T \right]. \quad (3)$$

This has the formal solution [4,9,10]

$$h_{\mu\nu}(\vec{x}, t) = 16\pi G \int d^3y \frac{\left[T_{\mu\nu}(\vec{y}, \tilde{t}) - \frac{1}{2} \eta_{\mu\nu} T(\vec{y}, \tilde{t}) \right]}{|\vec{x} - \vec{y}|}, \quad (4)$$

where \tilde{t} is the retarded time $\tilde{t} = t - |\vec{x} - \vec{y}|$. These are the gravitational analog of the Liénard–Wiechert potentials of ordinary electromagnetism, and the integral has support on the unperturbed backward light cone from the point \vec{x} .

In writing down this formal solution we have tacitly assumed that there is no incoming gravitational radiation. We have also assumed that the *global geometry* of spacetime is approximately Minkowski, a somewhat more stringent condition than merely assuming that the metric is *locally* approximately Minkowski. Finally note that the fact that we have been able to completely

gauge-fix Einstein gravity in a canonical manner is essential to argument. That we can locally gauge-fix to the Hilbert–Lorentz gauge is automatic. By the assumption of asymptotic flatness implicit in linearized Einstein gravity, we can apply this gauge at spatial infinity where the only remaining ambiguity, after we have excluded gravitational radiation, is that of the Poincare group. (That is: Solutions of the Hilbert–Lorentz gauge condition, which can be rewritten as $\nabla^2 x^\mu = 0$, are under these conditions unique up to Poincare transformations.) We now extend the gauge condition inward to cover the entire spacetime, the only obstructions to doing so globally coming from black holes or wormholes, which are excluded by definition. Thus adopting the Hilbert–Lorentz gauge in linearized gravity allows us to assign a *canonical* flat Minkowski metric to the entire spacetime, and it is the existence of this canonical flat metric that permits us to make the comparisons (between two different metrics on the same spacetime) that are at the heart of the argument that follows.

Now consider a vector k^μ which we take to be a null vector of the *unperturbed* Minkowski spacetime

$$\eta_{\mu\nu} k^\mu k^\nu = 0. \quad (5)$$

In terms of the full perturbed geometry this vector has a norm

$$||k||^2 \equiv g_{\mu\nu} k^\mu k^\nu \quad (6)$$

$$= h_{\mu\nu} k^\mu k^\nu \quad (7)$$

$$= 16\pi G \int d^3y \frac{T_{\mu\nu}(\vec{y}, \tilde{t}) k^\mu k^\nu}{|\vec{x} - \vec{y}|}. \quad (8)$$

Now assume the NEC

$$T_{\mu\nu} k^\mu k^\nu \geq 0, \quad (9)$$

and note that the kernel $|\vec{x} - \vec{y}|^{-1}$ is positive definite. Using the fact that the integral of a everywhere positive integrand is also positive, we deduce $g_{\mu\nu} k^\mu k^\nu \geq 0$. Barring degenerate cases, such as a completely empty spacetime, the integrand will be positive definite so that

$$g_{\mu\nu} k^\mu k^\nu > 0. \quad (10)$$

That is, a vector that is null in the Minkowski metric will be spacelike in the full perturbed metric. Thus the null cone of the perturbed metric must everywhere lie inside the null cone of the unperturbed Minkowski metric.

Because the light cones contract, the *coordinate* speed of light must everywhere decrease. (Not the *physical* speed of light as measured by local observers, as always in Einstein gravity, that is of course a constant.) This does however mean that the time required for a light ray to get from one spatial point to another must always increase compared to the time required in flat Minkowski

space. This is the well-known Shapiro time delay, and we see two important points: (1) to even define the delay (delay with respect to what?) we need to use the flat Minkowski metric as a background, (2) the fact that in the solar system it is always a delay, never an advance, is due to the fact that everyday bulk matter satisfies the NEC.

(We mention in passing that the strong energy condition [SEC] provides a somewhat stronger result: If the SEC holds then the proper time interval between any two timelike separated events in the presence of the gravitational field is always larger than the proper time interval between these two events as measured in the background Minkowski spacetime.)

Now subtle quantum-based violations of the NEC are known to occur [11], but they are always small and are in fact tightly constrained by the Ford–Roman quantum inequalities [12,13]. There are also *classical* NEC violations that arise from non-minimally coupled scalar fields [14], but these NEC violations require Planck-scale expectation values for the scalar field. NEC violations are never appreciable in a solar system or galactic setting. (SEC violations are on the other hand relatively common. For example: cosmological inflation, classical massive scalar fields, etc.)

From the point of view of warp drive physics, this analysis is complementary to that of [8], (and also to the comments by Coule [15], regarding energy condition violations and “opening out” the light cones). Though the present analysis is perturbative around Minkowski space, it has the advantage of establishing a direct and immediate *physical* connection between FTL travel and NEC violations. Generalizing this result beyond the weak field perturbative regime is somewhat tricky [1], and we have addressed this issue elsewhere. To even define effective FTL one will need to compare two metrics. (Just to be able to ask the question “FTL with respect to what?”).

Even if we simply work perturbatively around a general metric, instead of perturbatively around the Minkowski metric, the complications are immense: (1) the Laplacian in the linearized gravitational equations must be replaced by the Lichnerowicz operator; (2) the Green function for the Lichnerowicz operator need no longer be concentrated *on* the past light cone [physically, there can be back-scattering from the background gravitational field, and so the Green function can have additional support from *within* the backward light cone]; and (3) the Green function need no longer be positive definite.

For example, even for perturbations around a Friedman–Robertson–Walker (FRW) cosmology, the analysis is not easy [16]. Because linearized gravity is *not* conformally coupled to the background the full history of the spacetime back to the Big Bang must be specified to derive the Green function. From the astrophysical literature concerning gravitational lensing it is known that *voids* (as opposed to over-densities) can sometimes lead to a Shapiro time *advance* [17–19]. This is not in conflict with the present analysis and is not evidence for astrophysical NEC violations. Rather, because those calculations compare a inhomogeneous universe with a void to a homogeneous FRW universe, the

existence of a time advance is related to a suppression of the density below that of the homogeneous FRW cosmology. The local speed of light is determined by the local gravitational potential relative to the FRW background. Voids cause an increase of the speed of photons relative to the homogeneous background. The total time delay along a particular geodesic is, however, affected by two factors: the gravitational potential effect on the speed of propagation and the geometric effect due to the change in path of the photon (lensing) which may make the total path length longer. Thus traveling through a void doesn't *necessarily* imply an advance relative to the background geometry.

DISCUSSION

This note argues that any form of FTL travel requires violations of the NEC. The perturbative analysis presented here is very useful in that it demonstrates that it is already extremely difficult to even get even started: Any perturbation of flat space that exhibits even the slightest amount of FTL (defined as widening of the light cones) must violate the NEC. The perturbative analysis also serves to focus attention on the Shapiro time delay as a diagnostic for FTL, and it is this feature of the perturbative analysis we have extended elsewhere to the non-perturbative regime to provide both a non-perturbative definition of FTL [1], and a non-perturbative theorem regarding superluminal censorship.

REFERENCES

- [†] Electronic mail: visser@kiwi.wustl.edu
- [¶] Electronic mail: bruce@thphys.ox.ac.uk
- [§] Electronic mail: liberati@sissa.it
- 1. M. Visser, B. Bassett, and S. Liberati, *Superluminal censorship*, gr-qc/9810026.
- 2. M.S. Morris and K.S. Thorne, Am. J. Phys. **56**, 395 (1988).
- 3. M.S. Morris, K.S. Thorne, and U. Yurtsever, Phys. Rev. Lett, **61**, 1446 (1988).
- 4. M. Visser, *Lorentzian wormholes*, (AIP Press, New York, 1995).
- 5. D. Hochberg and M. Visser, Phys. Rev. Lett. **81**, 746 (1998); Phys. Rev. **D57**, 044021 (1998).
- 6. M. Alcubierre, Class. Quantum Grav. **11**, L73 (1994).
- 7. S.V. Krasnikov, Phys. Rev. **D57**, 4760 (1998).
- 8. K. Olum, Phys. Rev. Lett. **81**, 3567 (1998)
- 9. C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation*, (Freeman, San Francisco, 1973).
- 10. R.M. Wald, *General Relativity*, (Chicago University Press, 1984).
- 11. M. Visser, Phys. Lett. **B349**, 443 (1995); Phys. Rev. **D54**, 5103 (1996); **D54**, 5116 (1996); **D54**, 5123 (1996); **D56**, 936 (1997).

12. L.H. Ford and T.A. Roman, Phys. Rev. **D51**, 4277 (1995); **D53**, 5496 (1996)
13. M.J. Pfenning and L.H. Ford, Class. Quantum Grav. **14**, 1743 (1997);
14. E.E. Flanagan and R.M. Wald, Phys. Rev. **54**, 6233 (1996).
15. D.H. Coule, Class. Quantum Grav. **15**, 2523 (1998).
16. M.J. Pfenning and L.H. Ford, Phys. Rev. **D55**, 4813, (1997).
17. P. Schneider, J. Ehlers, and E.E. Falco, *Gravitational Lenses*, (Springer-Verlag, Berlin, 1992).
18. N. Mustapha, B.A. Bassett, C. Hellaby, and G.F.R. Ellis, Class. Quantum Grav. **15**, 2363 (1998).
19. G.F.R. Ellis and D.M. Solomons, Class. Quantum Grav. **15** 2381 (1998).